

Panel Presentation

NCSLI 2013 Session 6B

Easy Translation of TAR or TUR into Uncertainty

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Generic Uncertainty Expression



$$u(y) = \sqrt{u(D_n)^2 + u(C_n)^2}$$

where

 $u(D_n)$ is the standard uncertainty of the measurement

 $u(C_n)$ is the standard uncertainty of the calibration



Showing Next Level of Uncertainty



$$u(y) = \sqrt{u(D_n)^2 + u(C_n)^2}$$

$$= \sqrt{u(D_n)^2 + \left[u(D_{n-1})^2 + u(C_{n-1})^2\right]}$$



Assumptions

- Next, assume
 - TAR or TUR ≥4 at each level of calibration,
 - systems are in place that mitigate the effects of any potential sources of uncertainty not accounted for in the TAR or TUR being used, and
 - the numerator and denominator of the TAR or TUR are approximately known multiples of the associated standard uncertainties
- Example based on ANSI/NCSL Z540.3 TUR

$$TUR_{Z540.3} = \frac{\text{Upper Device Spec. - Lower Device Spec.}}{\text{Upper 95\% Cal. Unc. - Lower 95\% Cal. Unc.}}$$

$$\approx \frac{6 \cdot u(D_n)}{4 \cdot u(C_n)} = \frac{6}{4} \cdot \frac{u(D_n)}{u(C_n)} \equiv r_k \frac{u(D_n)}{u(C_n)}$$

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Relating Uncertainties at Different Levels



$$TUR \ge 4 \quad \Rightarrow \quad r_k \frac{u(D_n)}{u(C_n)} = r_k \frac{u(D_n)}{\sqrt{\left[u(D_{n-1})^2 + u(C_{n-1})^2\right]}} \ge 4$$

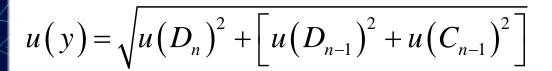
$$\Rightarrow \frac{r_k^2}{16}u(D_n)^2 \geq \left[u(D_{n-1})^2 + u(C_{n-1})^2\right]$$

$$\Rightarrow \frac{r_k^2}{16}u(D_n)^2 - u(D_{n-1})^2 - u(C_{n-1})^2 \ge 0$$

$$\Rightarrow \frac{r_k^2}{16}u(D_n)^2 - u(D_{n-1})^2 \ge 0$$



Put These Expressions Together ...



$$\leq \sqrt{u(D_n)^2 + \left\{\frac{r_k^2}{16}u(D_n)^2 - u(D_{n-1})^2\right\} + \left[u(D_{n-1})^2 + u(C_{n-1})^2\right]}$$

$$= \sqrt{u(D_n)^2 + \frac{r_k^2}{16}u(D_n)^2 + u(C_{n-1})^2}$$



... and Just Carry On ...

$$u(y) \le \sqrt{u(D_n)^2 + \frac{r_k^2}{16}u(D_n)^2 + u(C_{n-1})^2}$$

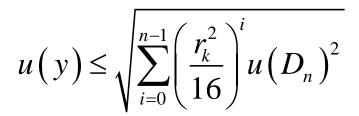
$$= \sqrt{u(D_n)^2 + \frac{r_k^2}{16}u(D_n)^2 + \left[u(D_{n-2})^2 + u(C_{n-2})^2\right]}$$

$$\leq \sqrt{u(D_n)^2 + \frac{r_k^2}{16}u(D_n)^2 + \left\{\left(\frac{r_k^2}{16}\right)^2 u(D_n)^2 - u(D_{n-2})^2\right\} + \left[u(D_{n-2})^2 + u(C_{n-2})^2\right]}$$

$$= \sqrt{u(D_n)^2 + \frac{r_k^2}{16}u(D_n)^2 + \left(\frac{r_k^2}{16}\right)^2 u(D_n)^2 + u(C_{n-2})^2}$$



... Until You End Up Here!



$$\leq \sqrt{\sum_{i=0}^{\infty} \left(\frac{r_k^2}{16}\right)^i u(D_n)^2}$$

$$= \sqrt{\frac{16}{16 - r_k^2}} \ u(D_n) \text{ if } \frac{r_k^2}{16} < 1$$

Now we can just use

$$u(y) = \frac{4(\text{Device Accuracy})}{\sqrt{16 - r_k^2} \sqrt{3}}$$

for legacy systems with TAR or TUR ≥4.

No further uncertainty analysis required!

Similar results appear to hold for systems based on EOPR as well.